Exercises, sheet 6.

1. Show that if T has Skolem functions then it eliminates quantifiers.
2. Use compactness and the infinitary Ramsey's theorem to prove the finitary version: for every finite n, k, l there is a natural number N such that for all sets V of cardinality $> N$ whose k-element subsets are coloured in l colours, there is a subset $V_0 \subseteq V$ of cardinality $\geq n$ which is monochromatic.
3. Solve Exercise 9.8.
4. Solve Exercise 9.9.
5. Convince yourself that Theorem 9.10 works for any linear order in place of $(\mathbb{N}, <)$.
6. For cardinals $\kappa, \mu, \nu, \lambda$ we write $\kappa \to (\mu)^\nu_\lambda$ if for every colouring of ν-sized subsets of κ using λ-many colours, there is a monochromatic subset of κ of size at least μ. Prove that $(2^\kappa)^+ \to (\kappa^+)^2_\kappa$.
7. Prove the Erdős–Rado theorem: for an infinite cardinal κ and a natural number $r \geq 0$ we have $\exp_r(\kappa)^+ \to (\kappa^+)^{r+1}_\kappa$, where $\exp_r(\lambda) = 2^{\exp_{r-1}(\lambda)}$ and $\exp_0(\lambda) = \lambda$.