Exercises, sheet 7.

1. Prove that an isolated type in a complete theory is realised.
2. Show that if a prime model exists, it is atomic.
3. Determine prime models of ACF_p, DLO_0. Do these have prime models over parameter sets? What are those in the cases they exist?
4. Show that if T is a complete theory with Skolem functions, then the prime model over any parameter set A exists.
5. Show that for a complete theory T is a countable language L, T has a prime model if and only if for every n isolated types are dense in $S_n(\varnothing)$.
6. A theory T in a countable language L is called *small* if for every n, $S_n(\varnothing)$ is countable. Show that a small theory has a prime model.
7. Show that if T in a countable language L is not small, then it has $2^{\aleph_0}$ isomorphism classes of models of cardinality $\aleph_0$ (the maximal possible cardinality).